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A comment on the *Aries* observations

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Dr Swallow's observations of the vertical profile of currents show how different a view of the deep ocean is now emerging. The tendency of currents to *increase* with depth below the permanent thermocline is so striking that it seems timely to point out that just this possibility was suggested recently by linearized theory (Rhines 1970). The work showed how short, yet quasi geostrophic, Rossby-topographic waves have a dominant mode that ceases to be independent of depth, and instead has faster currents near the bottom, due to the combined action of rotation, stratification and bottom slopes.

This comment is not the first (nor probably the last) application of linear theory to such observations, for both Longuet-Higgins (1965) and Phillips (1966) have suggested their interpretation as Rossby waves of a constant-depth ocean model. Noting this vertical shear, Phillips was forced to invoke a mixture of the familiar baroclinic and barotropic modes. There are, however, objections to this, since those baroclinic modes are exceedingly slowly propagating, and cannot be expected to persist with such strong currents. Also, as Dr Swallow points out, the current directions do not vary greatly in the vertical, suggesting an explanation as a single mode rather than a mixture. The bottom slopes, however, can greatly alter both horizontal propagation and vertical stucture. In particular the presence of vertical shear no longer implies impossibly small frequencies.

It may seem at first an unpleasant complication to have to consider all the above features simultaneously, but it could actually simplify the observational problem. Imagine, for instance, trying to test the dispersion relation of familiar depth-independent Rossby waves with an 'antenna' of current meters. Their anisotrophy, slowness and large scale make this a very difficult task, one that is not easy even with short surface waves at sea.

Consider, then, the advantage of having waves whose frequency depends only on the direction of propagation, but not their scale, and whose vertical extent varies directly with the horizontal scale. This is indeed the case in the short-wavelength limit of the bottom-trapped waves, and a single vertical chain of current meters, alone, might describe them completely.

For a simple description of the dynamics, consider the result of displacing the fluid horizontally over a bottom that slopes uniformly and gently. The β-effect alone would provide a restoring frequency $\approx fL/R$, where f is the Coriolis parameter, L the horizontal scale and R the radius of the Earth. Simple vortex stretching by the topography, alone, would contribute an amount $\approx f|\nabla H|\,L/H$, where H is the depth. But motion up the slope disturbs the density field as well, and this effect yields a frequency equal to the component of N, the buoyancy frequency, along the inclined particle paths, i.e. $\approx N|\nabla H|$. The analysis verifies that the character of the waves depends simply on which of these three frequencies dominates. The transition from Rossby waves to depth-independent topographic waves therefore occurs at $|\nabla H|\,R/H \approx 1$, but if

$$\frac{NH}{fL} > 1+\frac{H}{R|\nabla H|},$$

the buoyancy effect is most important, the waves become intensified at the bottom with vertical shear obeying the thermal wind equation and have a frequency given by the reduced component of N. Dominance of this effect requires that the waves be shorter than the internal Rossby deformation radius, NH/f, or 30 to 100 km (wavelengths of 200 to 600 km) in the oceans. The familiar depth-independent waves still apply to larger phenomena, but Dr Swallow has shown how small the dominant scale may be, even in mid-ocean.

The above inequality says that bottom slopes as gentle as the slope equivalent to β (1.4×10^{-3} at Bermuda) can alter the dynamics. The mean slope beneath the float trajectories is from one to three times this value and thus is sufficient. For a specific estimate, we take as an average value $N/f = 15$ cycles per half pendulum day. The observed vertical scale is about 4 km which gives a predicted horizontal scale, $4N/f$, of 60 km (the theory shows exponential decrease in current above the bottom for N a constant, but Dr Schmitz has reported privately calculations for realistic $N(z)$ that rather resemble Swallow's figure 4*b*; the appropriate N is that found by weighting $N(z)$ with the vertical eigenfunction). Taking an average bottom slope to be 2.5×10^{-3}, the buoyancy period $2\pi(N|\nabla H|)^{-1}$ is 27 days, and so even over such gradual slopes this mode can be more vigorous than are baroclinic Rossby waves. The floats, however, were observed to cross the depth contours at an angle less than $\frac{1}{2}\pi$, which reduces the buoyancy effect by the sine of this angle. In fact the trajectories in figure 4*a* deviate only slightly from the contours, but Crease (1962), dealing with the same data, showed that 2 weeks earlier, the currents were directed to the southeast, almost normal to the contours, and 6 weeks before that were running northward again (Crease also noted the bottom-intensification of the currents). Clearly ageostrophic excursions were taking place. For a mean angle of, say, $\frac{1}{6}\pi$ the characteristic period becomes two months.

The predicted 60 km scale and 1–2 month period, as well as the peculiar vertical structure, show that the effects in the theory were indeed competitive in the events described by Swallow and Crease. Theory alone gives little hint of the distribution of energy among various horizontal scales; one expects natural internal scales, as well as the externally applied stress and boundary scales, to appear. But the observed bottom intensification in five out of six samples and the measured spatial variance both indicate a dominant scale of the order of the internal deformation radius, NH/f. The successful use of density sections spaced 30 km apart verifies that the scale is at least this large.

Perhaps a candid conclusion is that we hope to have contributed a grain of truth, for an estimate of nonlinear effects is less kind to the theory. If one calculates the steepness of the waves (particle excursion divided by half-wavelength) as a measure of nonlinearity, it is of order unity. The motions are thus near the transition between advective eddies and waves. It is likely, however, that vestiges of linear theory will reach into this range, for the basic features of heavy fluid being forced up a slope, and quasi geostrophic potential vorticity being conserved, are still there. Of course this aspect ratio $H/L \approx f/N$ occurs in a wide range of rotating, stratified problems in which vertical motions are imposed at a boundary. Recent observations, for example, over the Great Meteor Seamount (Meinke 1971) suggest that fluid forced up onto the seamount enters into anticyclonic rotation, yet with velocities decreasing upwards, as if the imposed vortex compression had indeed been limited by density effects.

Bottom slopes may renew our interest in classical sections, for a characteristic of these waves is that their energy divides nearly equally between potential and kinetic. The elusive wavenumber spectrum might be estimated as a filtered version of the potential energy spectrum; the waves make less and less contribution to the potential energy as they become longer than NH/f. The observations suggest the dominance of rather small horizontal scales, but we should not be fooled into ignoring large-scale events. Local distortions of the velocity field may obscure larger coherences of a wave-function like pressure.†

We should remember that these are events of the deep ocean. The uppermost several hundred metres often moves more vigorously and, if Woods Hole's site D is typical, the upper region shows little coherence with long period motions below; it is as if the tops of our eddies and waves were literally being blown off.

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† During the meeting a film was shown which also emphasized small-scale waves. Computer-generated streamline patterns showed the evolution of a simple circular vortex, set into motion over various kinds of bottom topography. The propagation of vorticity was demonstrated as was the devastating effect of very rough, small-scale bumps. It was emphasized that the wavenumber spectrum (or 'scale') of the flow is not invariant in time, as soon as the medium has spatial variations. Even purely linear topographic waves enter into slow or rapid excursions in scale. A large vortex can fragment into topographic-scale eddies.

It is planned to submit stills of the film to *Reviews of Geophysics* and *Space Physics*.